Magnetic skyrmions in the conformal limit

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Magnetism and magnets

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- one of the oldest natural phenomena in the history of science
- however, deeply rooted in quantum mechanics => understanding only begins to emerge in the 20th century: <u>electron spin</u>

Paul Dirac, Werner Heisenberg, '20s - '30s

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Magnetism and magnets





Loose and Random Magnetic Domains

Magnetic Materials



Effect of Magnetization Domains Lined-up in Series

Repel

images borrowed from:

mammothmemory.net

Tom Whyntie. (2016), zenodo.com

- spins act as tiny magnetic dipoles
- quantum-mechanical interaction between spins: exchange
- in transition metals below the *critical temperature*, exchange results in local spin alignment into the *ferromagnetic state*
- magnetic field mediates long-range attraction/repulsion between magnets



Magnetic domains

- stray field
 frustrates the
 ferromagnetic
 order
- gives rise to a great variety of spin textures
- principle of pole avoidance

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J. McCord, J. Phys. D: Appl. Phys. 48, 333001 (2015)



Birth of nanomagnetism

The Nobel Prize in Physics 2007





Photo: U. Montan Albert Fert Prize share: 1/2

Photo: U. Montan Prize share: 1/2



Peter Grünberg

The Nobel Prize in Physics 2007 was awarded jointly to Albert Fert and Peter Grünberg "for the discovery of Giant Magnetoresistance"

paved the way to a new discipline in applied physics: **spintronics**





Terris'09

Piramanayagam'07

Explosion in magnetic hard disk drive capacity



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E. Grochowski, 2012 HDD capital equipment and technology report, Coughlin Associates

Non-volatile computer memories



magnetoresistive random access memory (MRAM):





racetrack memory:



"hard disk drive" without moving parts

C. Chappert et al., Nature Mat. 6, 813 (2007)

Next generation materials

atomically thin multilayers with strong spin-orbit coupling (SOC):







spin spirals and chiral domain walls:



2ML Fe on W(110)



Pd/Fe bilayer on Ir(111)



K. von Bergmann et al., J. Phys.: Condens. Matter 26, 394002 (2014)

magnetic skyrmion's:

Néel-type skyrmion



Bloch-type skyrmion









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C. Hanneken et al., Nature Nanotechnol. 10, 1039–1042 (2015)

Skyrmions on the track

Motion of a skyrmion in a track (from simulations)

40 nm





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Micromagnetic modeling framework

continuum theory (statics):

 $\Omega \subseteq \mathbb{R}^2$ - film shape

film thickness d = 0.6 nm

lateral dimension:

 $L \sim 100 \text{ nm}$

$$E(\mathbf{M}) = \frac{A}{M_{\mathrm{s}}^2} \int_{\Omega \times (0,d)} |\nabla \mathbf{M}|^2 \,\mathrm{d}^3 r + \frac{K}{M_{\mathrm{s}}^2} \int_{\Omega \times (0,d)} |\mathbf{M}_{\perp}|^2 \,\mathrm{d}^3 r - \mu_0 \int_{\Omega \times (0,d)} \mathbf{M} \cdot \mathbf{H} \,\mathrm{d}^3 r + \mu_0 \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \frac{\nabla \cdot \mathbf{M}(\mathbf{r}) \nabla \cdot \mathbf{M}(\mathbf{r}')}{8\pi |\mathbf{r} - \mathbf{r}'|} \,\mathrm{d}^3 r \,\mathrm{d}^3 r' + \frac{Dd}{M_{\mathrm{s}}^2} \int_{\Omega} (\bar{M}_{\parallel} \nabla \cdot \bar{\mathbf{M}}_{\perp} - \bar{\mathbf{M}}_{\perp} \cdot \nabla \bar{M}_{\parallel}) \,\mathrm{d}^2 r.$$

M, Slastikov, 2016

Here $\mathbf{M} = (\mathbf{M}_{\perp}, M_{\parallel}), \quad \mathbf{M}_{\perp} \in \mathbb{R}^2 \quad M_{\parallel} \in \mathbb{R} \quad |\mathbf{M}| = M_{\mathrm{s}} \text{ in } \Omega \times (0, d) \subset \mathbb{R}^3$

Parameters and their representative values:

- exchange constant $A = 10^{-11}$ J/m

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- anisotropy constant $K = 1.25 \times 10^6 \text{ J/m}^3$
- saturation magnetization $M_s = 1.09 \times 10^6 \text{ A/m}$
- DMI strength $D = 1 \text{ mJ/m}^2$ applied field strength $\mu_0 H = 100 \text{ mT}$

exchange length $\ell_{ex} = 3.66 \text{ nm}$

Need reduced micromagnetic models

analytically, the full 3D problem poses a formidable challenge:

- vectorial
- nonlinear
- nonlocal
- multiscale
- topological constraints

need a simplified model which is valid for the relevant parameter range and still captures quantitatively the physical features of the system

Solution: introduce reduced thin film models that are amenable to analysis

Use the tools from *rigorous asymptotic analysis* of calculus of variations



Dimension reduction

$$\mathbf{m} = (\mathbf{m}_{\perp}, m_{\parallel})$$

<u>assume</u> the magnetization $\mathbf{m} = \mathbf{M}/M_s$ does not vary significantly across the film thickness, measure lengths in the units of ℓ_{ex} , scale energy by Ad

$$E(\mathbf{m}) = \int_{\mathbb{R}^2} \left\{ |\nabla \mathbf{m}|^2 + (Q-1)|\mathbf{m}_{\perp}|^2 - 2\kappa \,\mathbf{m}_{\perp} \cdot \nabla m_{\parallel} \right\} d^2 r$$

+ $\frac{1}{2\pi\delta} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \left(\frac{1}{|\mathbf{r} - \mathbf{r}'|} - \frac{1}{\sqrt{|\mathbf{r} - \mathbf{r}'|^2 + \delta^2}} - 2\pi\delta^{(2)}(\mathbf{r} - \mathbf{r}')\delta \right) m_{\parallel}(\mathbf{r})m_{\parallel}(\mathbf{r}') d^2 r \, d^2 r'$
+ $\delta \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} K_{\delta}(|\mathbf{r} - \mathbf{r}'|) \nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}) \, \nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}') \, d^2 r \, d^2 r'$

Here:

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$$Q = \frac{2K}{\mu_0 M_s^2}, \qquad \kappa = D \sqrt{\frac{2}{\mu_0 M_s^2 A}}, \qquad \ell_{ex} = \sqrt{\frac{2A}{\mu_0 M_s^2}}, \qquad \delta = \frac{d}{\ell_{ex}}$$

$$K_{\delta}(r) = \frac{1}{2\pi\delta} \left\{ \ln\left(\frac{\delta + \sqrt{\delta^2 + r^2}}{r}\right) - \sqrt{1 + \frac{r^2}{\delta^2}} + \frac{r}{\delta} \right\} \simeq \frac{1}{4\pi r} \qquad \delta \ll 1$$

C. Garcia-Cervera, Ph.D. thesis (1999)

$$\mathbf{m} = (\mathbf{m}_{\perp}, m_{\parallel})$$

regime $\delta \ll 1$:

Taylor-expand in Fourier space

$$E(\mathbf{m}) \simeq \int_{\mathbb{R}^2} \left\{ |\nabla \mathbf{m}|^2 + (Q-1)|\mathbf{m}_{\perp}|^2 - 2\kappa \mathbf{m}_{\perp} \cdot \nabla m_{\parallel} \right\} d^2 r + \frac{\delta}{4\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{\nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}) \nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2 r \, d^2 r' - \frac{\delta}{8\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{(m_{\parallel}(\mathbf{r}) - m_{\parallel}(\mathbf{r}'))^2}{|\mathbf{r} - \mathbf{r}'|^3} d^2 r \, d^2 r'$$

M, Slastikov, 2016

the expression for the stray field energy is rigorously justified via Γ-expansion Knüpfer, M, Nolte, 2019

for bounded 2D samples, extra boundary terms appear

Di Fratta, M, Slastikov (in preparation)

proper definition of the non-local terms is via Fourier:

$$\frac{1}{2} \int_{\mathbb{R}^2} |\mathbf{k}| \left| \widehat{m}_{\parallel}(\mathbf{k}) \right|^2 \frac{d^2 k}{(2\pi)^2} = \frac{1}{8\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{(m_{\parallel}(\mathbf{r}) - m_{\parallel}(\mathbf{r}'))^2}{|\mathbf{r} - \mathbf{r}'|^3} d^2 r \, d^2 r', \qquad \Big\} \begin{array}{l} \text{surface charges} \\ \frac{1}{2} \int_{\mathbb{R}^2} \frac{|\mathbf{k} \cdot \widehat{\mathbf{m}}_{\perp}(\mathbf{k})|^2}{|\mathbf{k}|} \frac{d^2 k}{(2\pi)^2} = \frac{1}{4\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{\nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}) \nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, d^2 r \, d^2 r'. \\ \Big\} \begin{array}{l} \text{volume charges} \\ \text{charges} \end{array}$$

Rescaling of energy

$$m = (m', m_3)$$

 $m' = (m_1, m_2)$

rescale

$$\bar{x} := \frac{Q-1}{\kappa+\delta}x \text{ and } \bar{m}(\bar{x}) := m\left(\frac{\kappa+\delta}{Q-1}\bar{x}\right)$$

<u>energy</u>:

$$E_{\sigma,\lambda}(m) = \int_{\mathbb{R}^2} |\nabla m|^2 \,\mathrm{d}x + \sigma^2 \bigg(\int_{\mathbb{R}^2} |m'|^2 \,\mathrm{d}x - 2\lambda \int_{\mathbb{R}^2} m' \cdot \nabla m_3 \,\mathrm{d}x + (1-\lambda) \left(F_{\mathrm{vol}}(m') - F_{\mathrm{surf}}(m_3) \right) \bigg)$$

parameters:

$$\sigma := \frac{\kappa + \delta}{\sqrt{Q - 1}}$$
 and $\lambda := \frac{\kappa}{\kappa + \delta}$

ultimately, we wish to consider the asymptotic limit $\sigma \to 0$ with λ fixed



conformal limit



for bubble skyrmion, the stray field energy *diverges* with radius:

$$F_{\text{surf}}(m_{R,3}) \sim R \ln R \qquad \qquad \text{M, Simon, 2019}$$
hence $m: \mathbb{R}^2 \to \mathbb{S}^2, \ \nabla m \in L^2, \ m' \in L^2 \ \neq \ E_{\sigma,\lambda}(m) > -\infty$
no hope to construct solutions as absolute minimizers
with prescribed degree Bogdanov, Yablonskii, 1989
contrast this with the local case Bogdanov, Kudinov, Yablonskii, 1989
Melcher, 2014: Li, Melcher, 2018



Compact skyrmions as local minimizers

introduce:

$$\mathcal{A} := \left\{ m \in \mathring{H}^1(\mathbb{R}^2; \mathbb{S}^2) : \int_{\mathbb{R}^2} |\nabla m|^2 \, \mathrm{d}x < 16\pi, \, m + e_3 \in L^2(\mathbb{R}^2), \, \mathcal{N}(m) = 1 \right\}$$

why 16π ? Topological lower bound:

 $m\in \mathring{H}^1(\mathbb{R}^2;\mathbb{S}^2)$

 $\mathcal{N}(m) := \frac{1}{4} \int m \cdot (\partial_1 m \times \partial_2 m) \, \mathrm{d}x$

 $\int_{\mathbb{R}^2} |\nabla m|^2 \, \mathrm{d}x \ge 8\pi \, |\mathcal{N}(m)| \qquad |\nabla m|^2 \pm 2m \cdot (\partial_1 m \times \partial_2 m) = |\partial_1 m \mp m \times \partial_2 m|^2$

allows to exclude splitting in the concentration compactness arguments

Theorem 1. Let $\sigma > 0$ and $\lambda \in [0,1]$ be such that $\sigma^2(1+\lambda)^2 \leq 2$. Then there exists $m_{\sigma,\lambda} \in \mathcal{A}$ such that

$$E_{\sigma,\lambda}(m_{\sigma,\lambda}) = \inf_{\widetilde{m}\in\mathcal{A}} E_{\sigma,\lambda}(\widetilde{m}).$$

adapting arguments of Melcher, 2014, and Döring and Melcher, 2017 see also Greco, 2019



not much info about the structure of the solutions

Conformal limit

$$E_{\sigma,\lambda}(m) = \int_{\mathbb{R}^2} |\nabla m|^2 \,\mathrm{d}x + \sigma^2 \bigg(\int_{\mathbb{R}^2} |m'|^2 \,\mathrm{d}x - 2\lambda \int_{\mathbb{R}^2} m' \cdot \nabla m_3 \,\mathrm{d}x + (1-\lambda) \left(F_{\mathrm{vol}}(m') - F_{\mathrm{surf}}(m_3) \right) \bigg)$$

setting $\sigma = 0$ leads to harmonic maps from \mathbb{R}^2 to \mathbb{S}^2 with prescribed degree complete solution formally obtained by Belavin and Polyakov, 1975 degree 1 minimizers of $F(m) := \int_{\mathbb{R}^2} |\nabla m|^2 \, dx$ over $m \in \mathring{H}^1(\mathbb{R}^2; \mathbb{S}^2)$ belong to: $\mathcal{B} := \{S\Phi(\rho^{-1}(\bullet - x)) : S \in SO(3), \rho > 0, x \in \mathbb{R}^2\}$

i.e., dilations, rotations and translations of:

$$\Phi(x) := \left(-\frac{2x}{1+|x|^2}, \frac{1-|x|^2}{1+|x|^2}\right)$$

see also: Eells and Sampson, 1964 Lemaire, 1978 Wood, 1974 Brezis, Coron, 1985

furthermore, if $\phi \in \mathcal{B}$ then $\int_{\mathbb{R}^2} |\nabla \phi|^2 \, \mathrm{d}x = 8\pi$ and

$$\Delta \phi + |\nabla \phi|^2 \phi = 0$$
 $\mathcal{N}(\phi) = 1$

and vice versa

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minimizers of $E_{\sigma,\lambda}$ as $\sigma \to 0$ are almost minimizers of F

=> minimizers are close to \mathcal{B}

Lin. 1999

Rigidity estimate for degree 1 harmonic maps

define the class of degree 1 Sobolev maps from \mathbb{R}^2 to \mathbb{S}^2

$$\mathcal{C} := \left\{ \tilde{m} \in \mathring{H}^1(\mathbb{R}^2; \mathbb{S}^2) : \mathcal{N}(\tilde{m}) = 1 \right\}$$

and the *Dirichlet distance* to degree 1 Belavin-Polyakov profiles:

$$D(m; \mathcal{B}) := \inf_{\widetilde{\phi} \in \mathcal{B}} \left(\int_{\mathbb{R}^2} \left| \nabla \left(m - \widetilde{\phi} \right) \right|^2 \, \mathrm{d}x \right)^{\frac{1}{2}}$$

Theorem 2. For every $m \in C$ there exists $\phi \in \mathcal{B}$ that achieves the infimum in the Dirichlet distance $D(m; \mathcal{B})$. Furthermore, there exists a universal constant $\eta > 0$ such that

$$\eta D^2(m; \mathcal{B}) \le F(m) - 8\pi.$$

Gustafson, Kang, Tsai, 2007

- conformal invariance of the harmonic maps => switch to maps from \mathbb{S}^2 to \mathbb{S}^2
- reduce the problem to that of stability of the identity map on $\,\mathbb{S}^2$

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- spectral gap for the linearized problem via vectorial spherical harmonics

Smith, 1975; Mazet, 1973; Davila, del Pino, Wei, 2019; Chen, Liu, Wei, 2020 Di Fratta, Slastikov, Zarnescu, 2019; Luckhaus, Zemas, 2019

From linear stability to rigidity

Lemma 3. There exists a universal constant $\tilde{\eta} > 0$ such that the following holds: Let $p \in [1, \infty)$. Then there exists a constant $C_p > 0$ such that if $m \in H^1(\mathbb{S}^2; \mathbb{S}^2)$ satisfies $\int_{\mathbb{S}^2} |\nabla(m - \mathrm{id}_{\mathbb{S}^2})|^2 \, \mathrm{d}\mathcal{H}^2 \leq \tilde{\eta}$, then we have the estimate

$$\left(\int_{\mathbb{S}^2} |m - \mathrm{id}_{\mathbb{S}^2}|^p \,\mathrm{d}\mathcal{H}^2\right)^{\frac{1}{p}} \leq C_p \left(\int_{\mathbb{S}^2} |\nabla(m - \mathrm{id}_{\mathbb{S}^2})|^2 \,\mathrm{d}\mathcal{H}^2\right)^{\frac{1}{2}}$$

Furthermore, there exists a universal constant C > 0 such that the Moser-Trudinger type inequality

$$\int_{\mathbb{S}^2} e^{\frac{2\pi}{3} \frac{|m - \mathrm{id}_{\mathbb{S}^2}|^2}{\|\nabla (m - \mathrm{id}_{\mathbb{S}^2})\|_2^2}} \,\mathrm{d}\mathcal{H}^2 \le C$$

holds.

Lemma 4. Let $\tilde{\eta} > 0$ be as in Lemma 3. For $m \in \mathcal{C}$ with $D^2(m; \mathcal{B}) < \tilde{\eta}$ we have $\left(\frac{2}{3} - \frac{2}{3}C_4^2 D(m; \mathcal{B}) - \frac{19}{12}C_4^4 D^2(m; \mathcal{B})\right) D^2(m; \mathcal{B}) \le F(m) - 8\pi,$

where C_4 is the constant from Lemma 3.

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Ansatz

every ansatz-free analysis requires a good ansatz $\stackrel{\bigcirc}{=} f$ problem: $\phi \in \mathcal{B} \Rightarrow E_{\sigma,\lambda}(\phi) = \infty !$

 $f(r) := \frac{2r}{1+r^2}$

anisotropy energy blows up logarithmically => need a suitable cutoff at infinity for L > 1, introduce

$$\Phi_L(x) := \left(-f_L(|x|)\frac{x}{|x|}, \operatorname{sign}(1-|x|)\sqrt{1-f_L^2(|x|)}\right) \qquad f_L(r) := \begin{cases} f(r) & \text{if } r \le L^{\frac{1}{2}}, \\ f\left(L^{\frac{1}{2}}\right)\frac{K_1(rL^{-1})}{K_1\left(L^{-\frac{1}{2}}\right)} & \text{if } r > L^{\frac{1}{2}}. \end{cases}$$

 K_1 is the modified Bessel function of the second kind of order 1

- decays exponentially

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 arises from the exact minimizers of the exchange + anisotropy at infinity



$S_{\theta} := \begin{pmatrix} \cos \theta & -\sin \theta & 0\\ \sin \theta & \cos \theta & 0\\ 0 & 0 & 1 \end{pmatrix}$

Upper bound on energy

fix
$$\rho > 0$$
, $\theta \in [-\pi, \pi)$, $L > 1$ and $S_{\theta} \in SO(3)$ as above define a test profile

$$\phi_{\rho,\theta,L}(x) := S_{\theta} \Phi_L(\rho^{-1}x)$$

then

N

$$E_{\sigma,\lambda}(\phi_{\rho,\theta,L}) \simeq 8\pi + \frac{4\pi}{L^2} + 4\pi\sigma^2\rho^2 \log\left(\frac{4L^2}{e^{2(1+\gamma)}}\right) - 8\pi\sigma^2\lambda\rho\cos\theta + \sigma^2(1-\lambda)\frac{\pi^3\rho}{8}\left(3\cos^2\theta - 1\right)$$

minimized by

$$\rho_{0} \simeq \frac{\bar{g}(\lambda)}{16\pi} \frac{1}{|\log \sigma|}, \quad L_{0} \simeq \frac{16\pi}{\bar{g}(\lambda)} \frac{|\log \sigma|}{\sigma}, \quad \theta_{0}^{\pm} := \begin{cases} 0 & \text{if } \lambda \ge \lambda_{c}, \\ \pm \arccos\left(\frac{32\lambda}{3\pi^{2}(1-\lambda)}\right) & \text{else.} \end{cases}$$

$$\lambda_{c} := \frac{3\pi^{2}}{32+3\pi^{2}}, \qquad \bar{g}(\lambda) := \begin{cases} (8+\frac{\pi^{2}}{4})\pi \lambda - \frac{\pi^{3}}{4} & \text{if } \lambda \ge \lambda_{c} \\ \frac{128\lambda^{2}}{3\pi(1-\lambda)} + \frac{\pi^{3}}{8}(1-\lambda) & \text{else} \end{cases}$$

$$\text{See also Komineas, Melcher, Venakides, 2020}$$

Main result

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Theorem 5. Let $\lambda \in [0,1]$. Let m_{σ} be a minimizer of $E_{\sigma,\lambda}$ over \mathcal{A} . Then there exist $x_{\sigma} \in \mathbb{R}^2$, $\rho_{\sigma} > 0$ and $\theta_{\sigma} \in [-\pi,\pi)$ such that $m_{\sigma} - S_{\theta_{\sigma}} \Phi(\rho_{\sigma}^{-1}(\bullet - x_{\sigma})) \to 0$ in $\mathring{H}^1(\mathbb{R}^2;\mathbb{R}^3)$ as $\sigma \to 0$, and $\lim_{\sigma \to 0} |\log \sigma| \rho_{\sigma} = \frac{\bar{g}(\lambda)}{16\pi}, \qquad \lim_{\sigma \to 0} |\theta_{\sigma}| = \theta_0^+,$ as well as $\lim_{\sigma \to 0} \frac{|\log \sigma|^2}{\sigma^2 \log |\log \sigma|} \left| E_{\sigma,\lambda}(m_{\sigma}) - 8\pi + \frac{\sigma^2}{|\log \sigma|} \left(\frac{\bar{g}^2(\lambda)}{32\pi} - \frac{\bar{g}^2(\lambda)}{32\pi} \frac{\log |\log \sigma|}{|\log \sigma|} \right) \right| = 0.$

$$\lambda_c := \frac{3\pi^2}{32 + 3\pi^2}, \qquad \bar{g}(\lambda) := \begin{cases} (8 + \frac{\pi^2}{4})\pi \lambda - \frac{\pi^3}{4} & \text{if } \lambda \ge \lambda_c \\ \frac{128\lambda^2}{3\pi(1 - \lambda)} + \frac{\pi^3}{8}(1 - \lambda) & \text{else} \end{cases}$$

<u>Remark</u>: quantitative estimate of the closeness to the BP profile available

Bernand-Mantel, M, Simon, 2020

$$\begin{split} E(\mathbf{m}) &\simeq \int_{\mathbb{R}^2} \left\{ |\nabla \mathbf{m}|^2 + (Q-1) |\mathbf{m}_{\perp}|^2 - 2\kappa \mathbf{m}_{\perp} \cdot \nabla m_{\parallel} \right\} d^2 r \\ &+ \frac{\delta}{4\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{\nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}) \nabla \cdot \mathbf{m}_{\perp}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d^2 r \, d^2 r' - \frac{\delta}{8\pi} \int_{\mathbb{R}^2} \int_{\mathbb{R}^2} \frac{(m_{\parallel}(\mathbf{r}) - m_{\parallel}(\mathbf{r}'))^2}{|\mathbf{r} - \mathbf{r}'|^3} d^2 r \, d^2 r' \end{split}$$

Interpretation



Outline of proof

- matching upper and lower bounds in terms of energies of truncated BP profiles in the spirit of Γ-equivalence
- use the established rigidity of degree 1 harmonic maps to estimate the remaining terms in the energy
- the main difficulty is that the limiting BP profile may not satisfy $\lim_{|x|\to\infty}\phi(x)=-e_3$
- estimate the anisotropy energy penalty for deviations of $\nu := \lim_{|x|\to\infty} \phi(x)$ from - e_3 , using our version of Moser-Trudinger inequality
- relate the difference between ν and $-e_3$ to the Dirichlet excess via relaxing the unit length constraint and minimizing the exchange + anisotropy
- conclude by utilizing the rigidity of the finite-dimensional energy of BP profiles



Skyrmion bags

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many more solutions in the homotopy classes (even w/o stray field):



Thank you for your attention!



Cosmonauts A.Balandin and G.Strekalov with the Banner of Peace. Mir Space Station.

